Research Plan in Future

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The blow-up analysis for elliptic partial differential equations involving Finsler Laplacian now becomes a fruitful subject in the qualitative and quantitative theory of partial differential equations. For example, see the work by G. Wang-C. Xia for the Finsler Liouville equations, and the work by C.Zhou- C.Zhou (2019, JFA) for the attainability and blow-up analysis for the maximizing sequence for the Trudinger-Moser inequality involving the Finsler Dirichlet norm.

In this proposition, I will concentrate on the large exponent problem involving Finsler Laplacian in two-dimension. This problem is strongly related to Finsler Liouville equation in two-dimension studied by Wang-Xia. Also, it has a deep connection with Finsler Trudinger-Moser inequality studied by Zhou-Zhou. For the isotropic case, the blow-up analysis for the positive solution u to $-\Delta u = u^p$ as $p \to \infty$ was initiated by J.Wei-X.Rei (1994, Trans. AMS) for the case that solutions are the least energy ones. The solution sequences exhibit typical concentration phenomena as $p \to \infty$ and Green's function of the domain determines the location of the concentration point and the asymptotic behavior of the normalized solution sequence. It is very natural to expect such concentration phenomena also will happen even if we replace the usual Laplacian by Finsler Laplacian. However, in the anisotropic case, we cannot rely on many mathematical tools, such as MMP or Kelvin transformations, which were useful for the isotropic case. On the other hand, the possible host researcher Professor has a research experience of this kind of large exponent problem with nonlinear boundary conditions. Though the situation is not quite similar, several techniques to bypass the usual tools for the Laplacian case will be helpful. In fact, in a collaboration work with Prof. Takahashi, I have already obtained some results for the least energy solutions of this problem. Results are organized in a paper and we have already submitted it to a journal and arXiv. Since the least energy solutions are most natural variational solutions, they have good properties compared to general solutions for the problem. However, even for the least energy solutions, there are many open problems, such as the uniqueness or nondegeneracy, which will be the next subject of the proposal. Also, the possible blow-up analysis for general solutions, not necessarily least energy, is widely open for anisotropic cases, and it seems an interesting topic in this field.

Both research proposals are new and need new techniques in analysis. Therefore, the success of these proposal will give a big effect to the current PDE community.